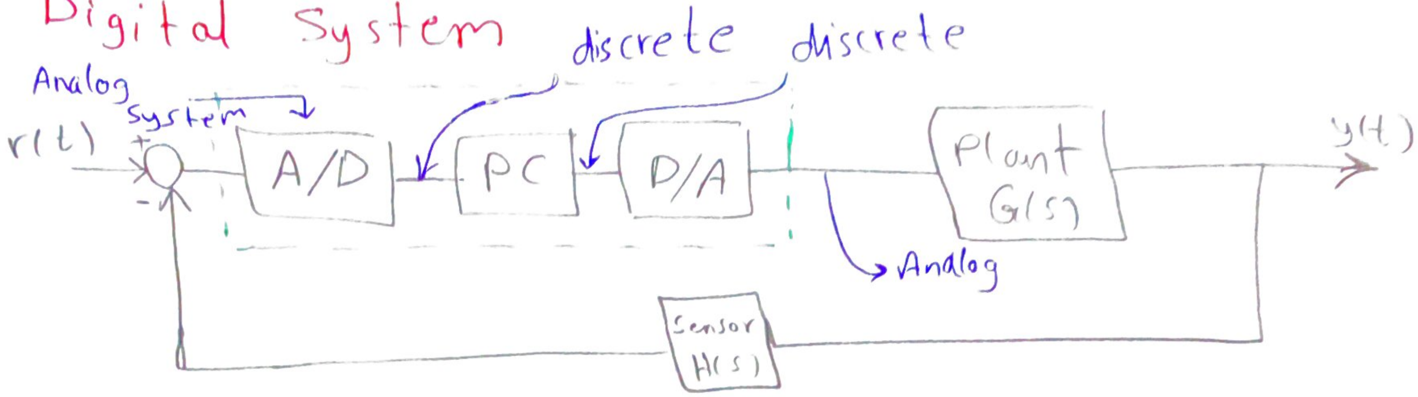
Digital

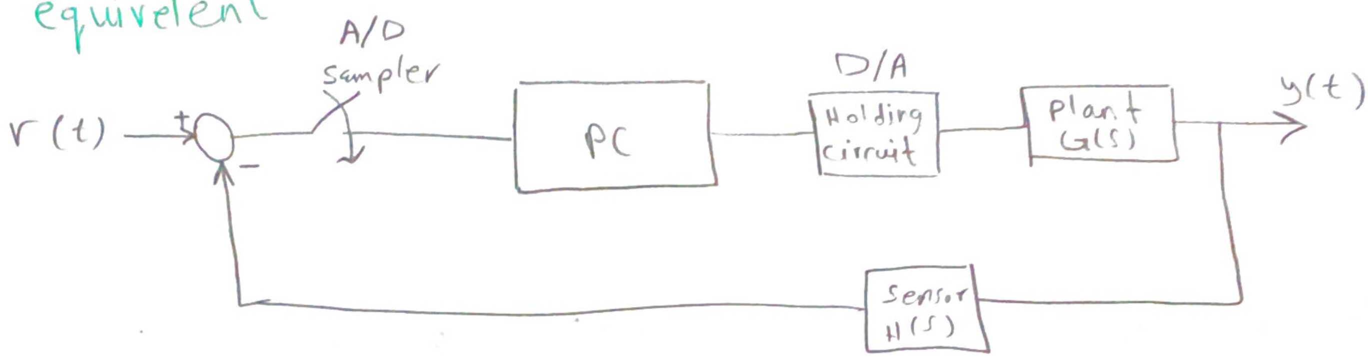
15/2/2016 الارشيني د عرفة محاضرة [1]

Revision on Z.T.

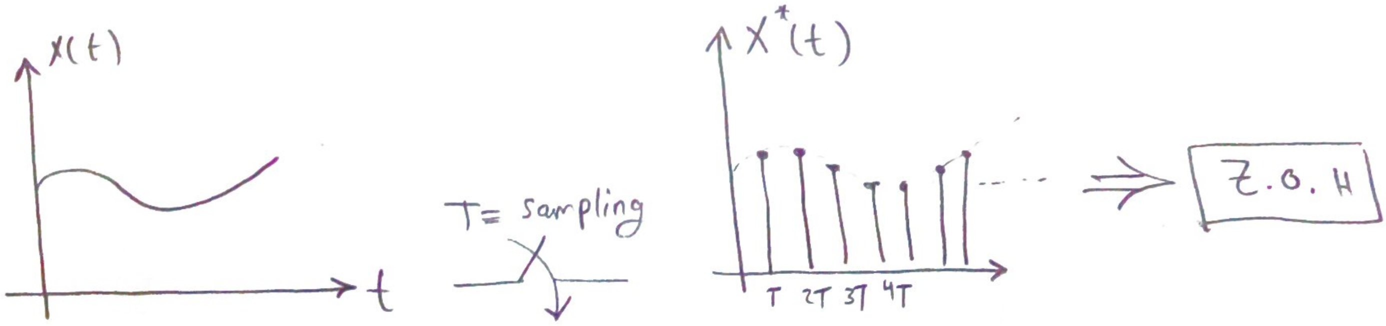
Digital System



equivelent

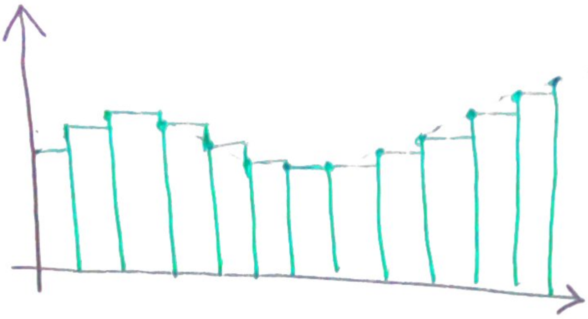


* Digital System
  1. A/D
  2. D/A converter = Holding Circuit
     - Z.O.H (Zero order Hold)
     - F.O.H (First order Hold)
     - S.O.H (second order Hold)



The output of the sampler is called Star signal
≡ Sampled Signal

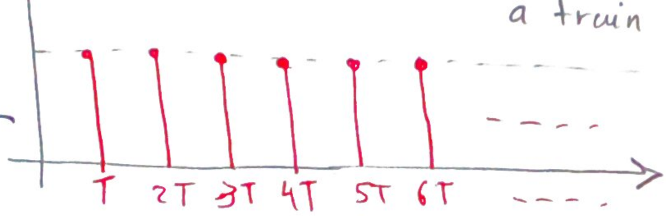
what Z.O.H does?



$\Leftarrow$ Can be smoothed using L.P.F

* to get more accurate approximation of the input signal, we need more samples $\Rightarrow$ requires higher frequency for sampling.

$\sum_{K=0}^{\infty} \delta(t-KT)$

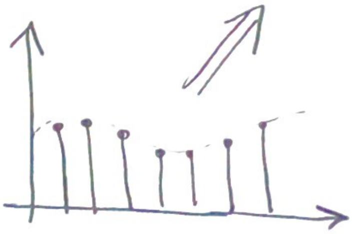


the sampler can be viewed as a train of impulses

$$x^*(t) = \sum_{K=0}^{\infty} x(t)\, \delta(t-KT) = x(0) + x(T) + x(2T) + \cdots$$

K $\longrightarrow$ sampling no.

T $\longrightarrow$ Sampling period



$$x^*(t) = \sum_{K=0}^{\infty} x(KT)\, \delta(t-KT)$$

L.T.

$$X^*(s) = \sum_{K=0}^{\infty} X(KT)\, e^{-KTs} \quad , \text{Let } z = e^{Ts}$$

$$\boxed{X(z) = X^*(s)\Big|_{e^{Ts}=z} = \sum_{K=0}^{\infty} X(KT)\, z^{-K}}$$

Ex (1)

$$x(t) = u(t) = 1$$

$$X(z) = \sum_{K=0}^{\infty} 1 . z^{-K} = \sum_{K=0}^{\infty} z^{-K} = 1 + z^{-1} + z^{-2} + \cdots$$

$$= \frac{1}{1-z^{-1}} = \frac{z}{z-1}$$

Ex (2)

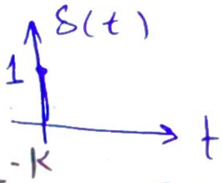
$$x(t) = e^{at}$$

$$X(z) = \sum_{K=0}^{\infty} e^{aKt} z^{-K} = 1 + e^{aT} z^{-1} + e^{2aT} z^{-2} + \cdots$$

$$= \frac{1}{1-e^{aT} z^{-1}} = \frac{z}{z-e^{aT}}$$

Ex (3)

$$x(t) = \delta(t)$$



$$X(z) = \sum_{K=0}^{\infty} \delta(KT) z^{-K} = \delta(0) + \cancel{\delta(1)}_{0} z^{-1} + \cdots$$

$$= 1 .$$

Ex (4)

$$x(t) = a^{t}$$

$$X(z) = \sum_{K=0}^{\infty} a^{KT} z^{-K} = 1 + a^{KT} z^{-1} + a^{2KT} z^{-2} + \cdots$$

$$= \frac{1}{1-a^{T} z^{-1}} = \frac{z}{z-a^{T}}$$

Ex (5)

$$x(t) = t$$

$$X(z) = \sum_{K=0}^{\infty} KT\, z^{-K} = T z^{-1} + 2T z^{-2} + 3T z^{-3} + \cdots$$

$$z\, X(z) = T + 2T z^{-1} + 3T z^{-2} + \cdots$$

$$z X(z) - X(z) = T + T z^{-1} + T z^{-2} + T z^{-3} + \cdots$$

$\Longrightarrow$ continue

$$zX(z) - X(z) = \frac{T}{1 - z^{-1}}$$

$$(z-1)\,X(z) = \frac{T}{1 - z^{-1}} = \frac{T\,z}{z - 1}$$

$$\boxed{X(z) = \frac{T\,z}{(z-1)^2}}$$

Ex (6)

$$x(t) = \sin \omega t$$

$$\sin \omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$\cos \omega t = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$X(z) = \sum_{K=0}^{\infty} \sin(\omega K T)\, z^{-K}$$

$$= \sum_{K=0}^{\infty} \left[\frac{e^{j\omega KT} - e^{-j\omega KT}}{2j}\right] z^{-K}$$

$$= \frac{1}{2j}\left[\left(\sum_{K=0}^{\infty} e^{j\omega KT} z^{-K}\right) - \left(\sum_{K=0}^{\infty} e^{-j\omega KT} z^{-K}\right)\right]$$

$$= \frac{1}{2j}\left[\left(1 + e^{j\omega T} z^{-1} + e^{j2\omega T} z^{-2} + \dots\right) - \left(1 + e^{-j\omega t} z^{-1} + e^{-j2\omega t} z^{-2} + \dots\right)\right]$$

$$= \frac{1}{2j}\left[\frac{z}{z - e^{j\omega T}} - \frac{z}{z - e^{-j\omega T}}\right]$$

$$= \frac{1}{2j}\left[\frac{z(z - e^{-j\omega T}) - (z - e^{j\omega T})}{(z - e^{j\omega T})\,(z - e^{-j\omega T})}\right]$$

$$= \frac{z}{2j}\left[\frac{-e^{-j\omega T} + e^{j\omega T}}{z^2 - z(e^{j\omega T} + e^{-j\omega T}) + 1}\right] \rightarrow \sin$$

$$\hookrightarrow * \tfrac{2}{2} \Rightarrow \cos$$

$$= \frac{z \sin(\omega T)}{z^2 - 2z\cos(\omega T) + 1}$$

Ex (7)

$$X(t) = \cos \omega t$$

$$X(z) = \frac{z\,(z-\cos(\omega T))}{z^2 - 2z\cos(\omega T) + 1}$$

$\boxed{1}$ $Z\left[f_1(t)\Big|_{t=kT} \pm f_2(t)\Big|_{t=kT} \right] = F_1(z) \pm F_2(z)$

$\boxed{2}$ $Z\left[e^{\pm aT} f(t)\Big|_{t=kT} \right] = F(z)\Big|_{z=z e^{\mp aT}}$

$\boxed{3}$ $Z\left[a f(t)\Big|_{t=kT} \right] = a F(z)$

$\boxed{4}$ $Z\left[a^t f(t)\Big|_{t=kT} \right] = F(z)\Big|_{z=\frac{z}{a^T}}$

$\boxed{5}$ $Z\left[t f(t)\Big|_{t=kT} \right] = -Tz \dfrac{dF(z)}{dz}$

$\boxed{6}$ initial value : (the value at $t=0$)

$$f(0) = \lim_{t\to 0} f(t) = \lim_{z\to\infty} F(z)$$

Final value : (the value at $t=\infty$

$$f(\infty) = \lim_{t\to\infty} f(t) = \lim_{z\to 1} (z-1)\, F(z)$$

$$= \lim_{z\to 1} (1-z^{-1})\, F(z)$$

$\frac{z-1}{z}$

$\boxed{7}$ shifting

① delay: $Z[x(k-n)] = z^{-n} X(z)$

② Advance : $Z[x(k+n)] = z^n X(z) - z^n x(0) - z^{n-1} x(1) - \dots - z\, x(n-1)$

ex:

$$Z[x(k+1)] = z\,X(z) - z\,x(0)$$

$$Z[x(k+2)] = z^2 X(z) - z^2 x(0) - z\,x(1)$$

$$Z[x(k+3)] = z^3 X(z) - z^3 x(0) - z^2 x(1) - z\,x(2)$$

Proof:-

$$Z[x(k+1)] = \sum_{k=0}^{\infty} x(k+1)\, z^{-k}$$

$$= x(1) + x(2) z^{-1} + x(3) z^{-2} + \cdots$$

$$= z \cdot z^{-1} \left[x(1) + x(2) z^{-1} + x(3) z^{-2} + \cdots \right]$$

$$= z \left[x(1) z^{-1} + x(2) z^{-2} + x(3) z^{-3} + \cdots \right]$$

$$= z \left[\underbrace{x(0) + x(1) z^{-1} + x(2) z^{-2} + \cdots}_{X(z)} - x(0) \right]$$

$$= z\,X(z) - x(0)$$

** you can prove that for other advances **

Ex: $y(k+2) + 3y(k+1) + 2y(k) = \delta(k)$

Z.T. ↓ if $y(0) = 0,\ y(1) = -1$, Solve for $y(k)$

$$z^2 Y(z) - z^2 y(0) - z\,y(1) + 3\left[z\,Y(z) - z\,y(0) \right] + 2\,Y(z) = 1$$

$$z^2 Y(z) + z + 3z\,Y(z) + 2\,Y(z) = 1$$

$$(z^2 + 3z + 2)\, Y(z) = 1 - z$$

$$\Rightarrow Y(z) = \frac{1-z}{z^2+3z+2} = \frac{1-z}{(z+1)(z+2)}$$

$$= \frac{A_1}{(z+1)} + \frac{A_2}{(z+2)}$$

$$A_1 = \frac{2}{1} = 2$$

$$A_2 = \frac{3}{-1} = -3$$

$$Y(z) = \frac{2\,z\,z^{-1}}{z+1} - \frac{3\,z\,z^{-1}}{z+2}$$

$I.Z.T = Z^{-1}.T. \Rightarrow y(K) = 2(-1)^{K-1} u(K-1) - 3(-2)^{K-1} u(K-1)$

Ex: $F(z) = \dfrac{z(z+1)}{(z+2)(z+4)}$, find $z^{-1}.T.$

$$F(z) = z\left[\frac{z+1}{(z+2)(z+4)}\right]$$

$$= z\left[\frac{A_1}{(z+2)} + \frac{A_2}{(z+4)}\right]$$

$A_1 = \frac{-1}{2}$

$A_2 = \frac{-3}{-2} = \frac{3}{2}$

$$F(z) = -0.5\,\frac{z}{z+2} + \frac{1.5\,z}{z+4} \Rightarrow z^{-1}.T \quad **$$

$$f(K) = -0.5\,(-2)^K + 1.5\,(-4)^K$$

Report: Solve the example for T = 0.5 sec

Hint: difference will be at **